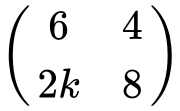
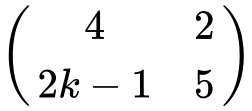
# 1.14 Introduction to matrices\_P\_1

**1a.** *[3 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

*2****A***=           ***(A1)***

2***A***− ***B*** =         ***A2   N3***

***[3 marks]***

**1b.** *[3 marks]*

## Markscheme

Evidence of using the definition of determinant                    ***(M1)***

Correct substitution                ***(A1)***

*eg* 4(5) − 2(2 − 1), 20 − 2(2 − 1), 20 − 4 + 2

det (2***A***− ***B***) = 22 − 4               ***A1  N3***

***[3 marks]***

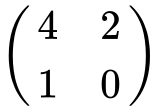
**2a.** *[2 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of addition     ***(M1)***

*eg* at least two correct elements

***A*** + ***B*** =     ***A1    N2***

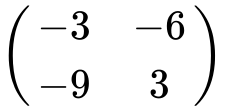
***[2 marks]***

**2b.** *[2 marks]*

## Markscheme

evidence of multiplication    ***(M1)***

*eg* at least two correct elements

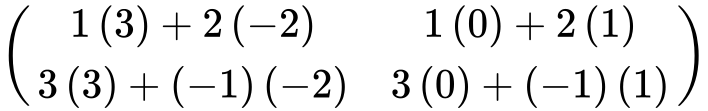
−3***A*** =     ***A1    N2***

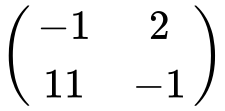
***[2 marks]***

**2c.** *[3 marks]*

## Markscheme

evidence of matrix multiplication (in correct order)   ***(M1)***

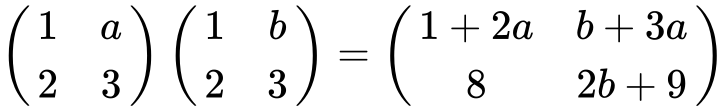
*eg*  ***AB*** = 

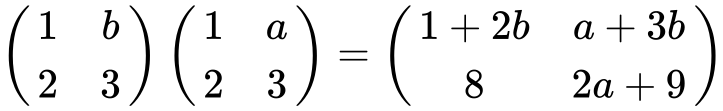
***AB*** =        ***A2  N3***

***[3 marks]***

**3a.** *[4 marks]*

## Markscheme



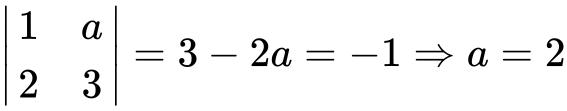
        ***M1A1***

So require         ***M1A1***

***[4 marks]***

**3b.** *[2 marks]*

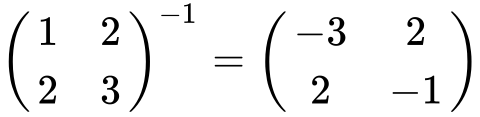
## Markscheme

       ***M1A1***

***[2 marks]***

**3c.** *[1 mark]*

## Markscheme

       ***A1***

***[1 mark]***

**4.** *[4 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

      ***(M1)(A1)***



       ***(A1)***

 or      ***(A1) (C4)***

**Note:** Both answers are required for the final ***(A1)***.

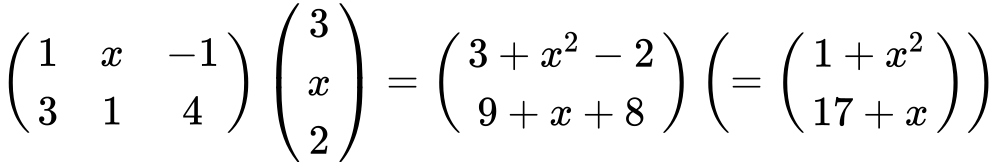
***[4 marks]***

**5a.** *[3 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Attempting to multiply matrices       ***(M1)***

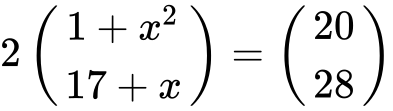
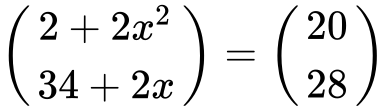
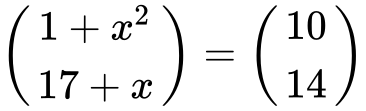
        ***A1A1    N3***

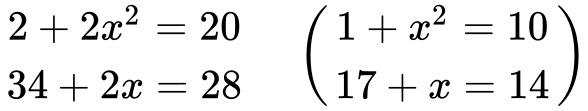
***[3 marks]***

**5b.** *[3 marks]*

## Markscheme

Setting up equation      ***M1***

*eg*  ,  ,  

            ***(A1)***

        ***A1    N2***

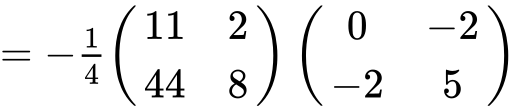
***[3 marks]***

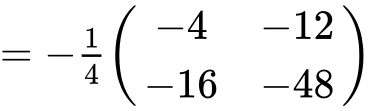
**6.** *[4 marks]*

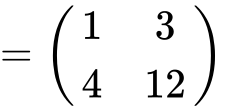
## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

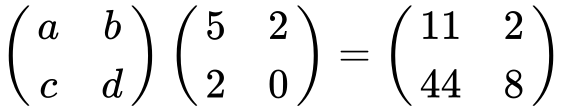
     ***(M1)***

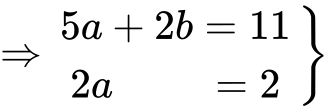
     ***(M1)***

      ***(A1)***

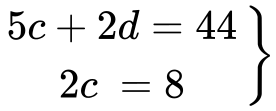
      ***(A1)***

**OR**

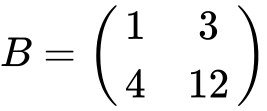
     ***(M1)***



,       ***(A1)***



,       ***(A1)***

     ***(A1)*** ***(C4)***

**Note:** Correct solution with inversion (ie *AB* instead of *BA*) earns FT marks, (maximum ***[3 marks]***).

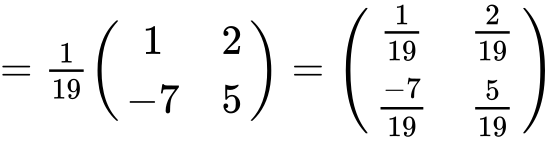
***[4 marks]***

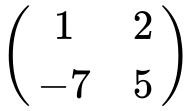
**7a.** *[2 marks]*

## Markscheme

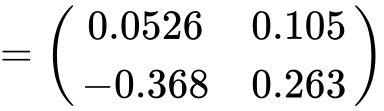
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

det ***A*** = 5(1) − 7(−2) = 19

***A***        ***(A2)***

**Note:** Award ***(A1)*** for , ***(A1)*** for dividing by 19.

**OR**

***A***                       ***(G2)***

***[2 marks]***

**7b.** *[2 marks]*

## Markscheme

***XA*** + ***B*** = ***C* ⇒** ***XA*** = ***C*** – ***Β***        ***(M1)***

***X*** = (***C*** – ***Β***)***Α***   ***(A1)***

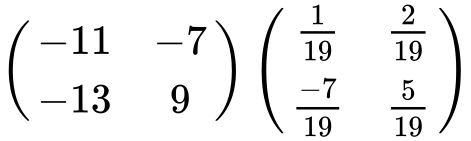
**OR**

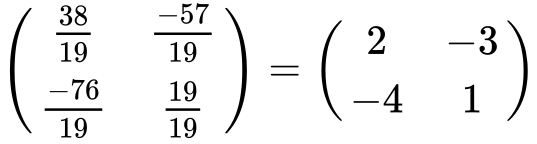
***X*** = (***C*** – ***B***)***A***      ***(A2)***

***[2 marks]***

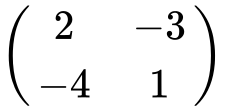
**7c.** *[2 marks]*

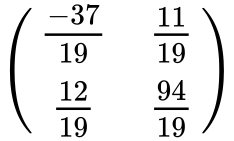
## Markscheme

(***C*** – ***Β***)***Α*** =        ***(A1)***

⇒ ***X*** =    ***(A1)***

**OR**

***X*** =       ***(G2)***

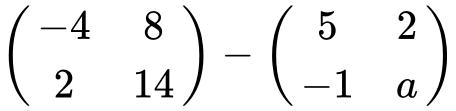
**Note:** If premultiplication by **A**is used, award ***(M1)(M0)*** in part (i) but award ***(A2)*** for  in part (ii).

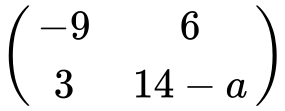
***[2 marks]***

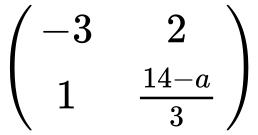
**8a.** *[3 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

3***Q*** =        ***(A1)***

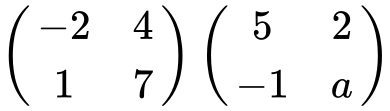
3***Q*** =        ***(A1)***

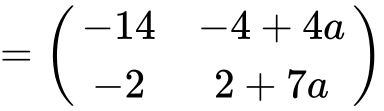
***Q*** =        ***(A1)  (N3)***

***[3 marks]***

**8b.** *[4 marks]*

## Markscheme

***CD*** = 

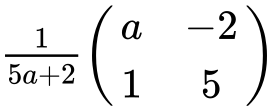
        ***(A1)(A1)(A1)(A1)  (N4)***

***[4 marks]***

**8c.** *[2 marks]*

## Markscheme

det ***D*** = 5 + 2   (may be implied)       ***(A1)***

***D***= ***(A1) (N2)***

***[2 marks]***

**9a.** *[1 mark]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

 = 5***A1  N1***

***[1 mark]***

**9b.** *[2 marks]*

## Markscheme

 + 9 = 4   ***(M1)***

 = −5   ***A1 N2***

***[2 marks]***

**9c.** *[3 marks]*

## Markscheme

Comparing elements  3(2) − 5() = −9         ***M1***

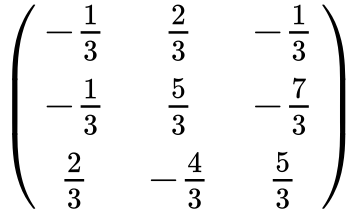
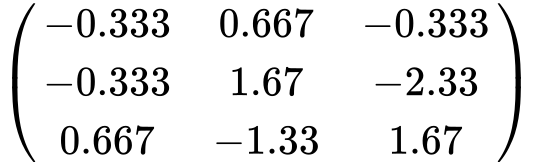
 = 3      ***A2 N2***

***[3 marks]***

**10a.** *[2 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

***A*** =  or           ***A2 N2***

***[2 marks]***

**10b.** *[1 mark]*

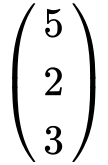
## Markscheme

***X*** = ***AB***        ***A1 N1***

***[1 mark]***

**10c.** *[3 marks]*

## Markscheme

***X*** =         ***A3 N3***

***[3 marks]***

**11a.** *[1 mark]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

 = 4       ***A1 N1***

***[1 mark]***

**11b.** *[1 mark]*

## Markscheme

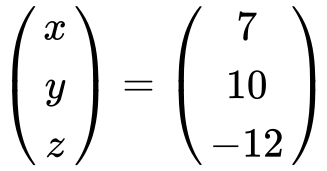
 = 7       ***A1 N1***

***[1 mark]***

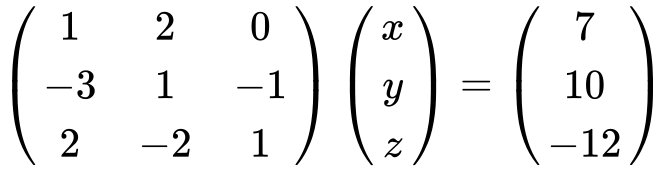
**11c.** *[1 mark]*

## Markscheme

**EITHER**

***A***      ***A1 N1***

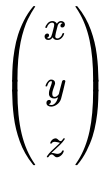
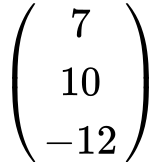
**OR**

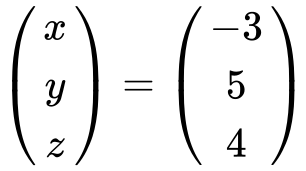
      ***A1 N1***

***[1 mark]***

**11d.** *[3 marks]*

## Markscheme

 = ***A***      (accept algebraic method)         ***(M1)***

     (accept  = −3,  = 5,  = 4)        ***A2    N3***

***[3 marks]***

**12a.** *[7 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

suppose *f*(***X***) = *f*(***Y***) , ie ***AX*** = ***AY***      ***(M1)***

then ***AAX*** = ***AAY      A1***

***X*** = ***Y      A1***

since *f*(***X***) = *f*(***Y***) ⇒ ***X*** = ***Y***, *f* is an injection      ***R1***

now suppose **C** ∈ ***M*** and consider *f*(***D***) = ***C*** , ie ***AD*** = ***C      M1***

then ***D*** = ***A*** ***C*** (***A*** exists since ***A*** is non- singular)      ***A1***

since given **C** ∈ ***M***, there exists ***D*** ∈ ***M*** such that *f*(***D***) = ***C*** , *f* is a surjection      ***R1***

therefore *f* is a bijection      ***AG***

***[7* marks]**

**12b.** *[4 marks]*

## Markscheme

suppose *f*(***X***) = ***Y***, ie ***AX*** = ***Y***      ***(M1)***

then det(***A***)det(***X***) = det(***Y***)      ***A1***

since det(***A***) = 0, it follows that det(***Y***) = 0      ***A1***

it follows that *f* is not surjective since the function cannot reach non-singular matrices      ***R1***

therefore *f* is not a bijection      ***AG***

***[4* marks]**

**13a.** *[3 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**  
***A*** = 4***A*** + 4A***I*** + ***I*** or equivalent       ***M1A1***  
= 4(2***A*** + ***I***) + 4***A*** + ***I***       ***A1***  
= 8***A*** + 4I + 4***A*** + ***I***  
= 12***A*** + 5***I***      ***AG***

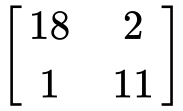
***[3 marks]***

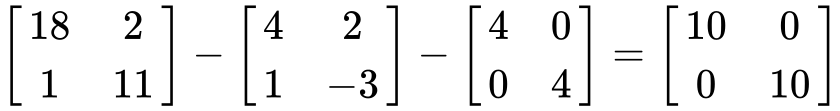
**METHOD 2**  
***A*** = ***A***(2***A*** + ***I***) = 2***A*** + ***AI*** = 2(2***A*** + ***I***) + ***A***(= 5***A*** + 2***I***)       ***M1A1***  
***A*** = ***A***(5***A***+ 2***I***)       ***A1***  
= 5***A*** + 2***A*** = 5(2***A***+ ***I***) + 2***A***  
= 12***A*** + 5***I***       ***AG***

***[3 marks]***

**13b.** *[3 marks]*

## Markscheme

***B*** =       ***(A1)***

      ***(A1)***

     ***A1***

***[3 marks]***

**14a.** *[8 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

\(\left( {\begin{array}{\*{20}{c}}  
 1 \\   
 3 \\   
 2

\end{array}\,\,\begin{array}{\*{20}{c}}

0 \\

1 \\   
 a

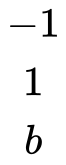
\end{array}\,\,\begin{array}{\*{20}{c}}

1 \\

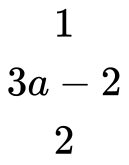
2 \\

{-1}

\end{array}\,\,



} \right) \Rightarrow \left( {



\,\,\begin{array}{\*{20}{c}}

0 \\   
 0 \\

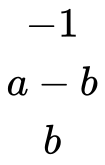
a  
\end{array}\,\,\begin{array}{\*{20}{c}}

1 \\

{2a+1} \\

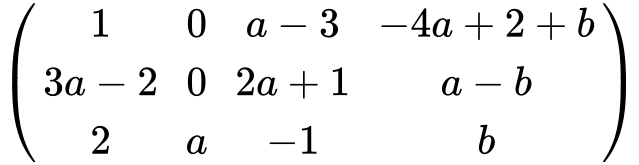
{-1}

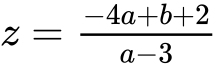
\end{array}\,\,



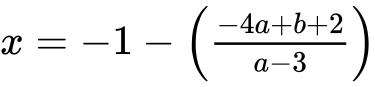
} \right)\) or equivalent

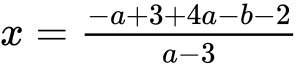
***M1A1***

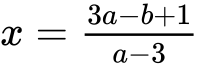


      ***M1A1***

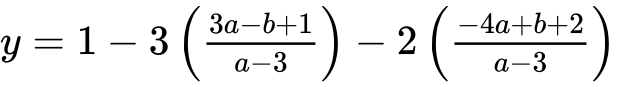
      ***M1***

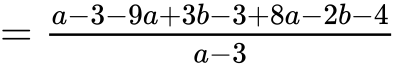


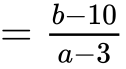


      ***A1***

       ***M1***





     ***A1***

***[8 marks]***

**14b.** *[1 mark]*

## Markscheme

when  = 3 the denominator of ,  and  = 0      ***R1***

**Note:** Accept any valid reason.

hence no unique solutions       ***AG***

***[1 mark]***

**14c.** *[4 marks]*

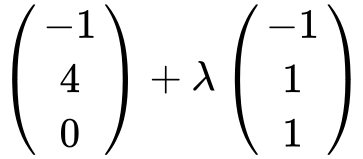
## Markscheme

For example let       ***(M1)***

      ***(A1)***



      ***(A1)***

*r* =       ***A1***

***[4 marks]***

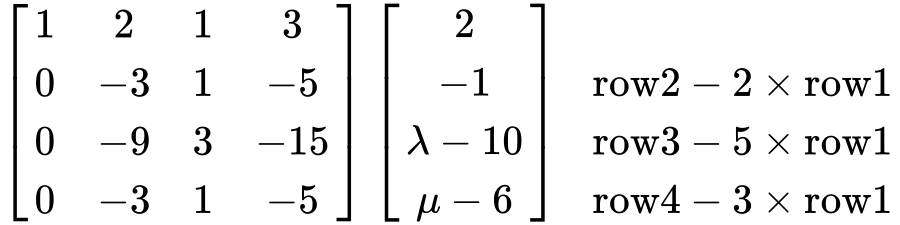
**Note**: Accept answers which let  or .

**15a.** *[5 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

using row operations on  matrix,     ***M1***

     ***A2***

or any alternative correct row reductions

**Note:**     Award ***A1*** for two correct row reductions.

     ***A1***

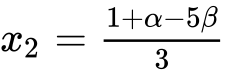
     ***A1***

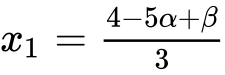
***[5 marks]***

**15b.** *[3 marks]*

## Markscheme

let      ***M1***

     ***A1***

     ***A1***

**Note:**     Alternative solutions are available.

***[3 marks]***

**15c.** *[2 marks]*

## Markscheme

the rank is 2     ***A1***

because the matrix has 2 independent rows or a correct comment based on the use of rref     ***R1***

***[2 marks]***

**16.** *[4 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(i)     ***M*** ***MM*** only exists if the number of columns of ***M*** equals the number of rows of ***M     R1***

hence ***M*** is square     ***AG***

(ii)     apply the determinant function to both sides     ***M1***

***M******M***

use the multiplicative property of the determinant

***M******M******M******M***     ***(M1)***

hence ***M*** or 1     ***A1***

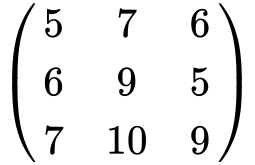
***[4 marks]***

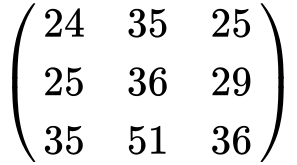
**17.** *[12 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(a)     successive powers of ***A*** are given by

***A***      ***(M1)A1***

***A***      ***A1***

it follows, considering elements in the first rows, that





     ***M1A1***

solving,     ***(M1)***

     ***A1***

**Note:**Accept any other three correct equations.

**Note:**Accept the use of the Cayley–Hamilton Theorem.

***[7 marks]***

(b)     (i)     it has been shown that

***A******A******A******I***

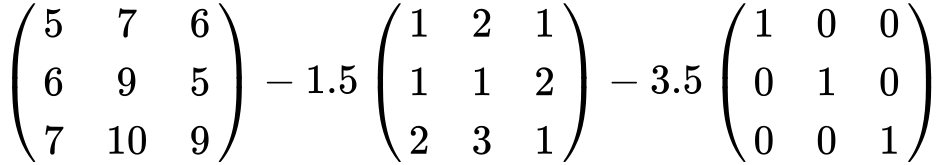
multiplying by ***A***,     ***M1***

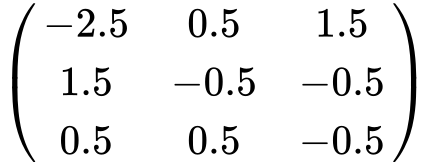
***A******A******I******A***     ***A1***

whence

***A******A******A*** ***I***     ***A1***

(ii)     substituting powers of ***A***,

***A***     ***M1***

=    ***A1***

**Note:**Follow through their equation in (b)(i).

**Note:**Line (ii) of (ii) must be seen.

***[5 marks]***

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